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PROBLEMS;

DESIGNED TO FACILITATE THE APPLICATION

OF

ALGEBRA TO GEOMETRY,

AND OF

THE PRINCIPLES OF NATURAL PHILOSOPHY

T O

PRACTICAL QUESTIONS.

PHILADELPHIA:

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GEOMETRICAL PROBLEMS.

- 1. Required the dimensions of an oblong garden, containing 3 acres, and bounded by 104 perches of pale fence.

 Ans. 40 P. by 12 P.
- 2. A gentleman has a garden 100 feet long and 80 feet broad, and a gravel walk is to be made of equal width half round it; what must be the width of the walk, so that it may take up just \(\frac{1}{4}\) of the ground?

 Ans. 11.8975 ft.
- 3. In a right-angled triangle, given the hypothenuse 75, and the sum of the other sides 105; to find the sides.

 Ans. 60 and 45
- 4. In a semicircle whose diameter is 30, a triangle is inscribed, the area of which is 180; what are its sides?

 Ans. 12 $\sqrt{5}$ and 6 $\sqrt{5}$.
- 5. Given the base of a triangle 12, the altitude 7, and the difference of the other sides 3; to find those sides.

 Ans. 7.895 and 10.895.
- 6. The sides of a rectangular farm are to one another as 4 to 5; and the area expressed in acres, is just two-thirds of the perimeter expressed in chains. Required the sides.

 Ans. 24 and 30.
- 7. The area of a rectangular field is $7\frac{1}{2}$ acres, and the length of the diagonal 50 perches: required the sides.

 Ans. 30 and 40.
- 8. How many acres are contained in a square meadow, the diagonal of which is 20 perches longer than either of its sides?

 Ans. 14 A., 2 R., 11 P.

- 9. If the peak of Teneriffe be $2\frac{1}{2}$ miles above the level of the ocean, how far might an object lying on the surface, be seen from its summit; the diameter of the earth being 7912 miles? Ans. 140 $\frac{1}{2}$ miles.
- 10. A man has a semicircular plot of ground, which he wishes to lay out as a garden. He desires to know what will be the area of the largest bed of a square form which he can place in it; the diameter of the semicircle being 20 feet.

 Ans. 80 ft.
- 11. A person on board a ship at sea observes the topsail of a vessel beyond the offing, or line of the horizon. What must be the distance between the vessels, if his height above the water be 30 feet, and that of the lowest part of the sail 70?

Ans. 16.9466 miles.

12. A circular garden is surrounded by a walk that takes up just $\frac{1}{4}$ the area. The width of the walk is 10 feet. What is the area of the garden?

Ans. 17501 square ft.

- 13. In a rectangular tract of land, containing 58 acres, 3 roods, 8 perches, the difference of the lengths of the sides is just equal to the difference between the lengths of the longer side and the diagonal; what are the sides?

 Ans. 84 and 112 perches.
- 14. A May-pole, whose height was 100 feet, standing on a horizontal plane, was broken by a blast of wind, and the top struck the ground 34 feet from the bottom of the pole; required the length of each part.

 Ans. 44.22 standing, and 55.78 broken.
- 15. The area of a triangular field is 133 acres, and two of its sides measure 51 and 70 chains; what must be the length of the third side? Ans. 52.3355 chains.
 - 16. A person undertook to pull down an old lib-

erty pole by fastening a rope to the top, and passing it under a pulley fixed on the ground. The pole unexpectedly breaking 25 feet from the ground, the top of the descending fragment struck exactly on the pulley. What was the height of the pole, the length of rope between its top and the pulley being 150 feet.

Ans. 119.3 ft.

17. In a rectangular field, the length of the diagonal is 16 chains. From one of the angles a line is drawn at right angles to the diagonal, which it cuts at the distance of 4 chains from the nearest angle—this line being produced to the adjacent side; required the contents of the four parts into which the rectangle is thus subdivided.

Ans. 8 $\sqrt{3}$, 24 $\sqrt{3}$, $\frac{8}{3}$ $\sqrt{3}$, and $\frac{88}{3}$ $\sqrt{3}$, sq. chs.

18. Wishing to know the height of a tree standing on the bank of a stream, I determined to measure it the next day. Unfortunately however, that very night a wind broke off the tree, at 40 feet from the bottom, the top struck the opposite bank and again broke 10 feet from the top; while the middle part, which still remained fast to the tree, swung down and struck the ground 20 feet nearer the foot of the tree, than the top struck; but before I could measure its length, it broke loose above, and was swept away by the current. What was the height of the tree?

Ans. 92.26 ft.

- 19. I observed a calf tied to a stake in a field. The length of the string was such, that if it had been 12 feet longer, the calf would have had just twice as much pasture. How much would it have had, if the string had been 12 feet shorter? Ans. 904.7808 sq. ft.
- 20. In a level garden there are two lofty firs, having their tops ornamented with gilt balls; one is 100 feet high, and the other 80, and they are 120 feet dis-

tant at the bottom. Now the owner wants to place a fountain in a right line between the trees, to be equally distant from their tops; what will be its distance from the bottom of each tree, and also from each of the balls,

Ans. dist. from top 109.6585 ft. from bottom of the first 45, of the second 75.

- 21. The area of a right-angled triangle is 160, and the sides are in continued proportion; what are the sides?

 Ans. 15.861, 20.175 and 25.663.
- 22. A man has a triangular piece of meadow, containing 21 acres, 2 roods, 16 perches, one corner of which is under water. This he wishes to have banked in from the stream. The ditcher to whom he applied wishes to know the lengths of the two sides which are to be banked, and to find this, ties a rope to a stake which is situated at the outer corner and projects above the surface of the water. He then takes the other end of the rope to the other corners successively, but before he can measure its length, it breaks in the middle; and the stake becoming loosened, is swept away by the current, with a part of the rope attached to it. All that he can tell is that one side is 6 chains longer than the other. The length of the third side is 30 chains. Pray! assist the poor ditcher, by calculating from these data the lengths of the sides. Ans. 24 and 18.
- 23. A man has a triangular piece of ground, the sides of which are 15, 20, and 25 yards respectively. In this he wishes to place a rectangular building on the longer side, whose corners shall touch the other sides. The area of the building is to be 75 square yards. What will be the sides?

Ans. 6 and 12½ yds.

24. There is a house of three equal stories high; now a ladder being raised against it, 20 feet distant

from the base of the building, reaches the top; whilst another ladder 12 feet shorter, raised from the same point, reaches only to the top of the second story. Required the height of the building.

 $\overline{\mathbf{A}}$ ns. 41.7 ft. nearly.

- 25. It is required to lay out $4\frac{1}{3}$ acres of land in a triangular form, so that the length of one side may be 15 chains, and the lengths of the other sides in the ratio of 2 to 3; what must be the lengths of those sides?

 Ans. 29.59 and 44.38, or 7.79 and 11.69.
- 26. A wealthy merchant promises his daughter, that if she will tell him the value of a rectangular plate of pure gold, she shall have it for her dowry. The diagonal of the rectangle is 18 inches, and the distance around it 48. What is the value, supposing the plate to be 2 inches thick, and a cubic inch of gold to be worth \$210?

 Ans. \$52920.
- 27. I have a garden of a triangular shape, containing just one acre. The longer side is 20 perches, and the other two are in the ratio of 2 to 3. What are the sides?

 Ans. 16.11 and 24.16 P.
- 28. In a right-angled triangle, given the base 100, and the distance from the summit of the perpendicular to the perpendicular let fall from the right angle on the hypothenuse 60, to find the other sides.

Ans. perpen. 89.8, and hypoth. 134.4.

- 29. If an equilateral triangle be described in a circle, the diameter of which is 150 feet, what will be the length of its side?

 Ans. 129.9 ft.
- 30. Wishing to ascertain the height of a bluff overhanging a neighbouring river, I took a small skiff and rowed out into the stream; then standing upright in the boat, I found the reflected top of the bluff was

just 10 feet distant from my feet; but upon rowing 100 feet nearer the promontory, it was only 8 feet distant. My eyes were 6 feet above the surface of the water; what is the height of the bluff?

Ans. 294 ft.

- 31. When the foot of a certain ladder is placed 12 feet from the wall of a house, the top just reaches the eaves; but if the ladder be moved 8 feet further from the building, the top falls 4 feet below the eaves; required the height of the eaves and the length of the ladder?

 Ans. height 34 ft., length 36.055 ft.
- 32. The hypothenuse of a right-angled triangle is 46 feet, and the angle at the base 30°: What are the other sides?

 Ans. 23 and 39.8372.
- 33. In taking a survey of a tract of land, it was necessary to know the elevation of a certain hill. The angle of its inclination to the horizon I found to be 15°, and the distance from the bottom to the top 20 chains. Having no book of tables with me, it was necessary to calculate the height without them. Required the result.

 Ans. 5.1764 chains.
- 34. It is required to lay out 5 acres of ground in a triangular form, to be bounded by 135 perches of fence; the length of one side is to be 50 perches; what must be the lengths of the other sides?

Ans. 33.3785 and 51.6215 P.

- 35. The base of a triangle is 20 and the difference of the other sides 5. If a perpendicular be let fall from the vertex to the base, the difference of the segments of the base multiplied by the longer side, will be equal to the square of the shorter. What are the sides?

 Ans. 8.9039 and 13.9039.
- 36. The angle of inclination of a certain hill is 18°, and the distance from its foot to the top is 10

chains; required to find the altitude of the hill without the tables. Ans. 3.09 ch.

- 37. Having a circular yard 150 feet in diameter, I wish to plant ten trees around it equally distant from one another; how far apart must they be?
 - Ans. 46.3525 ft.
- 38. What is the length of the side of a regular pentagon described in a circle, whose diameter is 150 feet?

 Ans. 88.17 ft.
- 39. A solid glass globe, 8 inches in diameter, was blown out into a hollow sphere, the thickness of whose shell is $\frac{1}{10}$ of an inch; required the present diameter.

 Ans. 29.2123 inches.
- 40. In a right-angled triangle, the sum of the base and perpendicular is 140; and of the base and hypothenuse 180: What are the sides?

 Ans. base 80, perpen. 60, hyp. 100.
- 41. In a triangle there is given the rectangle of the sides 195, the rectangle of the segments of the base 45, and the perpendicular 12; to find the sides.

 Ans. 13, 14, and 15.
 - 42. A father who designed to bless
 Two lovely daughters fair,
 Bequeathed to each a lot of land,
 One round, the other square.
 Just twenty pounds per acre,
 Was the value it is said;
 And the shillings, that encompassed each,
 For each exactly paid.
 Now if a shilling, in its breadth,
 Be just an inch; pray tell,
 Ye fond admirers of the fair,
 To each, what portion fell.
 Ans. the square tract 627 A. 1 R. 2 P.,
 and the round 492 A. 2 R, 24 P.

43. Kingston lies directly north of the city of New York, and is 27 miles further from it than Easton, which is situated exactly west of that city; now the distance between Easton and Kingston is 110 miles; at what distance are those towns respectively from New York?

Ans. Easton 63.1, and Kingston 90.1 m.

- 44. The chords of two acres of a circle, whose diameter is 20 feet, are 9 and 12 feet respectively; what is the chord of their sum?

 Ans. 17.916 ft.
- 45. A person would enclose a tract of land in a square form, and wishes his fence to be seven rails high, and each pannel to be half a perch in length; required the quantity he must enclose, so that the number of acres enclosed may be just equal to the number of rails in the fence?

 Ans. 501760 A.
- 46. A garden is to be laid out in the form of a regular octagon, so as to contain \(\frac{1}{2} \) an acre; what must be the length of each side, and the distance from the centre to each corner?

Ans. side, 4.07 P., and dist. to corner, 5.32 P.

- 47. Being in a room opposite to a window, the bottom of which was just the height of my eye, I observed that up the edge of a window I could see 48 courses of bricks in a wall on the opposite side of the street; but walking in a direct line 20 feet toward the window, I found I could see 72 courses. Required the height of the window, supposing the width of the street to be 36 feet, and a course of bricks equal to 3 inches.

 Ans. 6.2522 ft.
- 48. A gentleman has a square tract of land, the diagonal of which is 10 poles longer than its side; he would convert two-thirds of the area into a garden of an octagonal form; in the centre of which he wishes a

fish-pond in the form of an equilateral triangle, to contain just 5 square poles. Required the length of each side of the garden, also that of the pond.

Ans. side of garden 8.97, of the pond 3.398 P.

- 49. Given the perpendicular 24, the line from the vertical angle to the middle of the base 30, and the line bisecting that angle 25, to find the sides of the triangle?

 Ans. 66.427, 56.558, 28.416.
- 50. A person has a right-angled triangular lot of ground, the sides of which are 30, 40 and 50 chains respectively; this he divides among his four sons, A. B. C. and D. in such a manner that A's share is a rectangle on the longest side, whose height is 12 chains, B's is the triangle between A's share, and the right-angle; C's the corner between the least side and the rectangle; and D's the remainder. What is each son's share?

Ans. A's 30 A., B's 15 A., C's 5 A. 1 R. 24 P., and D's 9 A. 2 R. 16 P.

- 51. A person wishing to know the area of a rhomboidal field, and having no compass, measures the whole distance round it, which he finds to be 80 perches, and the two diagonals, which are 20, and 36.0555. What is the area? Ans. 1 A. 3 R. 20 P.
- 52. Within a rectangular garden, there is a circular fountain, whose circumference, is 28, 40, 60 and 52 yards distant from the four corners respectively. What is the diameter of the fountain? Ans. 20 yds.
- 53. The difference of the two legs of a right-angled triangle, whose area is double that of the inscribed circle, is 6 chains. Required the area of the triangle, and the diameter of the circle.

Ans. area of triangle 8 A. 0 R. 22 P. diameter of circle 7.2 chains nearly.

54. On a horizontal plane is a garden, which was surrounded by a very high wall; but the owner, thinking the wall too high, had its top cut off, by a plane oblique to the horizon, so as to leave the highest point 16, and the lowest only 8 feet high; the horizontal distance between these two parts is 60 feet; now, it is found that a ladder, with its foot in a certain place in the garden, will just reach the top of the wall all round. Required the length of the ladder, and the area of the garden.

Ans. length of ladder 32.6 ft. nearly, and area 2852.46 sq. ft.

55. In a triangular field containing 12 acres, the perpendicular to the longest side, makes the segments of the base 9 and 11 chains respectively. The land next the shortest side is valued at 50 dollars per acre, and the other at only 30; required to divide this between two persons, by a line perpendicular to the longest side, so that the value of each one's share may be the same.

Ans. The perpendicular will cut the base 8.3785 chains from the shorter side.

56. The lengths of three lines, drawn from the angles of a triangular field to the middle of the opposite sides, are 18, 24, and 30 chains, respectively. What are the sides?

Ans. 20, 28.84 and 34.18 chains.

- 57. The three sides of a triangle are in the ratio of the numbers 4, 6 and 7, respectively, and the line bisecting the greatest angle, equals 20: What are the sides?

 Ans. 22.866, 34.3 and 40.016.
- 58. The three sides of a triangle are in the ratio of 5, 7 and 8, and the line drawn from the middle of the greatest side to the opposite angle 36: Required the sides.

 Ans. 39.28, 54.99, 62.85.

- 59. A long horizontal beam, supported only at the settremities, having been broken in the middle, the broken ends struck the ground at the distance of 3 feet from each other; the beam was 12 feet above the ground: Required its length.

 Ans. 97 feet 6 inches.
- 60. A man has a triangular piece of ground, the base of which is 200 yards, the sum of the other sides 350, and the line from the vertex to the middle of the base 150: Quere, the lengths of the sides?

 Ans. 131.6987 and 218.3013 yards.
- 61. If an equilateral triangle, whose area is 10000 square feet, be surrounded by a walk of the same width throughout, and equal to the area of its inscribed circle: What will be the breadth of the walk?

 Ans. 11.7 feet, nearly.
- 62. Required the dimensions of a cone, its content being 282 cubic inches, and its slant height to the diameter of its base, in the ratio of 5 to 4.

Ans. slant height 12.25 inches, diameter of base 9.8 in.

63. Standing on the bank of a river, I observed that the top of a tree on the other side, directly opposite, was reflected from a point on the water, 6 feet from where I stood; but after walking 100 feet down the stream, the reflected top was 8 feet from me. Required the height of the tree, my eyes being 5 feet above the surface of the water.

Ans. 94.5 feet, nearly.

64. The longer side of a rectangular tract of land measures 36 chains, and the sum of the diagonal and shorter side 54 chains: Required the content.

Ans. 54 acres.

65. The sides of a rectangular tract of land, are in the ratio of 2 to 5, and it contains just as many

acres as there are chains in its perimeter. What are the sides? Ans. 28 and 78.

66. Required the radius of that circle, in which the side of a regular pentagon is 1.

Ans.
$$\sqrt{\left(\frac{5+\sqrt{5}}{10}\right)} = 0.851.$$

- 67. In a right-angled triangle, the lines drawn from the acute angles to the middle of the opposite sides, are 16 and 21, respectively: Required the sides.

 Ans. 15.706, 27.88 and 32.
- 68. Two sides of a triangle are in the ratio of 5 to 3, and if a perpendicular be let fall from the angle contained by these sides, upon the opposite side, the segments will be 12 and 15, respectively: What are the sides?

 Ans. 6.75 and 11.25.
- 69. In a triangular field, all the sides of which are equal, there stands a chesnut tree; the shortest distances of this tree from the three sides, are 9, 5 and 12 chains, respectively: Required the area.

Ans. 39 acres 0 roods 5 perches.

- 70. The three distances from an oak, growing in an open plain, to three visible corners of a square field, lying at some distance, are known to be 80, 60 and 80 chains in successive order: Quere the area of the field?

 Ans. 64 acres 2 roods.
- 71. The area of an equilateral triangle, whose base falls on the diameter, and its vertex in the middle of the arc of a semicircle, is equal to 100: What is the diameter of the semicircle?

 Ans. 26.3214.
- 72. What is the side of an equilateral triangle, whose area cost as much paving at 8d. per foot, as the palisading the three sides did at a guinea per yard?

 Ans. 72.7462 feet.

73. In a triangular garden there is inscribed a circular hedge of yew, which touches the three sides; in one side there is a door just at the point of contact, whose distance from each of the adjacent corners is 20 and 30 yards respectively, and if a perpendicular be let fall from the opposite angle to this side, the rectangle contained by the segments thus formed, will be 576. Required the sides and area of the garden; also the diameter of the circle.

Ans. sides are 30, 40 and 50; area, 600;

diameter, 20.

- 74. In a right-angled triangle, there are given the ratio of the sides 3 to 4, and the difference between the areas of its inscribed circle and inscribed square, 20.2825. Required the sides. Ans. 30, 40 and 50.
- 75. What is the area of a right-angled triangle, the hypothenuse of which is x^{3z} and the other two sides, x^z and x^{3z} chains, respectively? Ans. $16\frac{1}{2}$ p.
- 76. In a right-angled triangular field, the legs are $3x^z$ and x^{3z} and the line bisecting the right-angle, x^{2z} chains: What is the content in acres?

Ans. 18.802, or .097 A.

- 77. If three equal circles be described in a circle, whose diameter is 24, so as to touch each other, what will be the length of their radii.

 Ans. 5.569.
- 78. A person who has a circular yard 300 feet in diameter, wishes to lay it out in six equal circular flower beds, touching each other and the circumference: he also wants the part in the middle, which is enclosed by the beds, made into a pond; but before he digs it, he would like to know its area; what will it be?

Ans. diameter of beds, 110 ft.; area of interior, 23562 sq. ft.

79. Given the three sides of a triangle, equal to a, b and c; required the expression for the area.

Ans.
$$\frac{1}{4}$$
 $\sqrt{\left\{ (a+b+c).(a+b-c).(a-b+c). (a-b+c). ($

- 80. In the middle of a circular pond, whose diameter is 80 feet, stood a high pole; now, this pole having been broken, it was observed that the top struck the brink of the pond, and that again breaking 10 feet from the extremity, the middle part just reached the bottom of the pole: Required the height of the pole.

 Ans. 160 ft.
- 81. A person has a rectangular tract of land, 12 chains in length and 9 in breadth, divided into two equal parts by a fence running from corner to corner; one of which is estimated at \$100 per acre, and the other at \$60. He offers to give his son \$400 worth, if he will tell where a line must be drawn to cut off that quantity in the least number of acres, and at the same time be parallel to one end. At what distance from the end must the line be drawn, also how many acres will be cut off on each side of the diagonal?

Ans. distance 4.834 chains, 3 R. 20 P. at \$60; and 3 A. 1 R. 20 P. at \$100.

- 82. Given the hypothenuse of a right-angled triangle 87, and the perpendicular 60; if, to a line bisecting the angle opposite the perpendicular, a perpendicular be drawn from the other extremity of the hypothenuse, what will be the sides of the resulting triangle?

 Ans. 12.924, 32.311, and 34.8.
- 83. The sides of a given triangle, are 15, 20 and 25; it is required to find the sides of a rectangle in-

scribed within it, on the longer side, whose area shall be \frac{1}{4} that of the triangle.

Ans. 9.464 and 5.283, or 2.536 and 19.717.

84. Wishing to know the height of a hill, I took my station on the top of it, and found that the place where I stood was exactly on a level with the top of a tree standing at the foot of the hill. The situation of the tree, however, was such that I could not, by the usual means, ascertain its height. I therefore measured the distance from the place where I stood to the foot of the tree, (the intervening ground being a regular slope) and found it to be 128 yards. I also observed, when I had advanced 103 yards towards the tree, that the angle subtended by the tree, was bisected by a line passing through a knot on the trunk of it, which knot I found to be 32 feet from the ground. What was the height of the hill?

Ans. 71.985 ft.

- 85. Wishing to ascertain the height of a neighbouring mountain, and not having any instruments necessary for that purpose, I take the following method: from the foot of a tall tree, whose height I know to be 100 feet, I measure a distance of 250 feet, and find the top of the tree precisely in a range with the top of the mountain; but on going 100 feet nearer the tree, the top of the mountain is observed to range with a black knot on the side of the tree 663 feet from the ground. Required the height of the mountain.

 Ans. 400 ft.
- 86. Given the perimeter of a right-angled triangle 60, and the perpendicular from the right-angle to the hypothenuse 12; to find the sides.

Ans. 15.20 and 25.

87. In the right-angled corner of a triangular tract of land, the sides of which are equidifferent, it is re-

quired to lay out a square garden, whose area shall be just equal to one half of that contained by the longest side and two lines drawn from the acute angles to the corner of the square. What is the area of the garden? the perimeter of the tract being 36 chains.

Ans. 1 A. 0 R. 3.5 P.

PHILOSOPHICAL PROBLEMS.

- 1. A man in walking fast, steps about twice in a second; how far would he walk in an hour, if his steps are 30 inches long?
- 2. What time would a steamboat, sailing at the uniform velocity of 15 miles per hour, require to sail from New York to Liverpool, a distance of 3000 miles?
- 3. Nuttall says, "that pigeons killed near New York, have been found with their crops full of rice, collected from the plantations of Carolina or Georgia; and as this kind of food is digested by them entirely in 12 hours, they must have travelled 300 miles in 6 hours:" with what velocity did they fly?
- 4. A ship weighing 336000 lbs., is dashed against a rock in a storm, with a velocity of 16 miles per hour, with what momentum did she strike? The momentum of 1 lb. moving 1 foot per second, being considered unity?
- 5. Two railway cars have their quantities of matter as 7 to 3, and their momenta as 8 to 5; what is the ratio of their velocities?

 Ans. 24 to 35.
- 6. There are two bodies, the greater is 8 times as heavy as the less, and moves with 48 times the force; what is their comparative velocities?

 Ans. 6 to 1.

- 7. Suppose the battering-ram of Vespasian weighed 5760 lbs., and was sufficient, when propelled with a certain velocity, to demolish the walls of Jerusalem; and suppose that a 32 pound ball fired with a velocity of 2000 feet per second, is found capable of doing the same execution, what was the velocity of the battering-ram?

 Ans. 11.11 ft. per second.
- 8. If a grain of light moving at the rate of 192.000 miles per second, were to strike directly against a mass of ice, moving at the rate of 1.45 feet per second, what weight of ice will the light stop?

Ans. 99878 lbs., or nearly 44½ tons.

- 9. How much would a 56 lb. weight weigh at the centre of the earth; and how much at a point half way from the centre to the surface?
- 10. How much weight would a rock that weighs 10 tons at the level of the sea, lose, if elevated to the top of a mountain 5 miles high?

 Ans. 55.8952 lbs.
- 11. After observing a flash of lightning, it was 12 seconds before the thunder was heard. Required the distance of the cloud whence it came?

Ans. 13704 feet, or nearly 24 miles.

- 12. How long after a cannon is fired in Philadelphia, will it be before the report is heard at Burlington, a distance of 20 miles?

 Ans. 92.46 seconds.
- 13. Perceiving a man at a distance hewing down a tree with an axe, I remarked that 6 pulsations passed between seeing him strike and hearing the report of the blow. What was the distance between us, allowing 70 pulsations to a minute?

Ans. 1 mile 198 yards.

- 14. After a flash of lightning, I observed that the rumbling of the thunder continued 10 seconds. What was the length of the flash, supposing the fluid to have passed between two horizontal clouds, and directly from me?

 Ans. 11420 ft.
- 15. How much less would a person weigh in a pit one mile deep, than he would at the surface?

 Ans. 3948 part of his weight.
- 16. If a ball $\frac{1}{10}$ of a mile in diameter, and of the same density as the earth, were placed at the distance of $\frac{1}{10}$ of a mile above it, what space would the earth move through to meet it, the diameter of the earth being taken 8000 miles?

Ans. 80.808,080.808, of an inch.

- 17. The quantity of matter in the sun being 355,000 times that of the earth, and the distance between them 95,000,000 miles; if the two bodies were abandoned to their mutual attraction, where would they meet?

 Ans. 2673 miles from the sun.
- 18. What would a man, who weighs 150 lbs. at the surface of the earth, weigh at the distance of the moon? (240,000 miles.)

 Ans. ½ of a lb.
- 19. If two lamps, one of which is 2 feet from an object, and the other 12, cast shadows of it of equal darkness, how much more light does one give than the other?

 Ans. 36 times as much.
- 20. If a piece of tobacco pipe ignited by the flame of an oxy-hydrogen blow-pipe 100 feet from an object, casts as deep a shadow as a candle at the distance of 1 foot, what is the ratio of the quantity of light?

 Ans. 10000 to 1.
- 21. Supposing our distance from the sun to be 95,000,000 of miles, and the distance of the planet

Venus, 69,000,000, what is the light and heat of Venus as compared with that of the earth?

Ans. 1.895 times as much.

- 22. How far will an iron ball fall in 5 seconds, in 10, in 60, in 19, in 45?*
- 23. How far will it fall in the 5th; in the 11th; in the 35th second?
- 24. What velocity will it acquire in 5 seconds, in 10, in 19, in 25, in 60, in 5 minutes, in 1 day?
- 25. With what velocity must it be thrown upward, in order to ascend a mile? Ans. 581.3 ft. per second.
- 26. A body fell from the top of a cliff 340 feet high; how far did it fall in the last 3 seconds?

 Ans. 298.957 ft.
- 27. If a stone were dropt into a pit 300 feet deep, how long would it require to reach the bottom?

 Ans. 4.33 seconds.
- 28. In what time would a cannon ball fall from a balloon three miles high, and what velocity would it acquire? Ans. time = 31.464 sec. vel. = 1006.85 ft. per second.
- 29. A musket ball shot directly upwards, returned in 14 seconds; how high did it ascend? Ans. 784 ft.
- 30. Suppose a body be projected downwards with a velocity of 18 feet per second, how far will it fall in 18 seconds?

 Ans. 5508 ft.

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^{*}The distance described in the first second, by a body falling from a state of rest, is $16\frac{1}{12}$ feet; but in order to render the calculations less laborious in these questions, it has been taken 16 feet; and no allowance has been made for the resistance of the air.

- 31. If a ball be projected upward with a velocity of 1000 feet per second, how high will it ascend?

 Ans. 15,625 ft.
- 32. If it fall 1000 feet in the last second, how far must it have fallen?

 Ans. 16,129 ft.
- 33. If a ball be shot horizontally, from a cannon 16 feet from the ground, with a velocity of 10,000 feet per second, how far will it be thrown?

 Ans. 10,000 ft.

34. If the above cannon were 4 miles from the ground, how far would the ball be thrown?

Ans. 68.808 miles, nearly.

35. A pistol ball having been shot downward from a balloon two miles high, is found to reach the earth in 7 seconds: required its velocity on leaving the pistol, and also on reaching the ground?

Ans. on leaving, 1,396‡ ft; at ground, 1620‡.

36. If a stone be thrown vertically, with a velocity of 144 feet per second, how high will it rise, and how far will it move in the first two seconds?

Ans. it will rise 324 ft; in 2 seconds, 224 ft.

- 37. A ball having been projected upward with sufficient velocity to carry it a mile high, it is required to find the space described in the first half of the time it is ascending?

 Ans. 3960 ft.
- 38. A body has been falling 12 seconds down an inclined plane, whose length is 2½ times its height; what velocity will it have acquired at the end of its fall?

 Ans. 153.6 ft. per sec.
- 39. The length of an inclined plane is 360 feet, and the height 80. If a body fall from a state of rest, from the top of the plane, what space will it pass

through in 8 seconds; what time will it be in falling 300 feet; and what velocity will it have on reaching the bottom? Ans. 227f ft. in 8 sec.; time, 9.185 sec.; velocity, 71.55 ft. per sec.

40. In St. Petersburg, it is said that the Russians amuse themselves by sliding down artificial hills, or inclined planes covered with ice. Now suppose two inclined planes of this description are placed opposite each other, one 48 feet high and 300 feet long, and the other 130 feet long, with the height unkown; it is observed, however, that a sled in descending the larger plane, acquired a velocity just sufficient to carry it to the top of the other. It is required to determine the height of the plane, and the time the sled would be in passing from the summit of one plane to the summit of the other?

Ans. height 48 ft.; and time 15.516 sec.

41. What is the length of a pendulum vibrating seconds at Philadelphia, where a body falls from a state of rest 16 1/2 feet the first second?

Ans. 39.11 inches.

42. The length of the seconds pendulum being 39.11 inches, what are the lengths of pendulums vibrating \(\frac{1}{2}\) and \(\frac{1}{2}\) seconds, and how long must a pendulum be to vibrate once in an hour?

Ans. \(\frac{1}{2}\) sec. 9.77\(\pi\) in.; \(\frac{1}{2}\) sec., 2.444 in.; and hour, 7999 miles.

- 43. A pendulum which vibrated seconds at the level of the sea, was found to vibrate but 3597 times in an hour, on the top of a neighbouring mountain; required the height of the mountain, the diameter of the earth being 7912 miles.

 Ans. 3\frac{1}{2} miles nearly.
- 44. What weight hung feet from the centre of motion in a steelyard, will balance a weight of 5000

lbs. suspended $\frac{1}{2}$ an inch on the other side of the centre?

Ans. $5\frac{3}{2}$ lbs.

- 45. Supposing a man's arm to be 28 inches long, and the tendon which raises it, fastened 1½ inches from the shoulder; what force must the tendon exert to support 50 lbs. in the hand, when at right angles to the body?

 Ans. 800 lbs.
- 46. If two boys carry a weight of 100 lbs. on a hand barrow between them, what will each carry, if the barrow be 8 feet long, and the weight be 3 feet from one end?

 Ans. 62½ and 37½ lbs.
- 47. In a lever of the first kind, if 500 lbs. 10 inches from the fulcrum, balance 69 lbs.; at what distance from the fulcrum must the last weight be?

Ans. 72.46 inches.

48. In a lever of the second kind of 20 feet in length, how far from the fulcrum must a weight of 180 lbs. be placed so that it may be supported by a power equal to 60 lbs. at the extremity?

Ans. 62 ft.

- 49. A cylindrical lever is 16 feet long, and weighs 12 lbs.; its longer arm is 9 feet, and its shorter 7; at the extremity of the shorter arm a weight of 4 lbs. is suspended; what weight will be required at the end of the other arm to balance it?

 Ans. 17 lbs.
- 50. If, in a pair of fraudulent scales 20 inches long, the arm from which the weight is suspended, be 9 inches, what would a purchaser lose on a pound of tea worth \$1.37½ cts. per pound; and what would he gain by changing the weight into the other scale?

 Ans. loss 25 cts. and gain 30%.
- 51. A stick 40 inches long having been nicely balanced in the middle; at 3 inches from one end were

- placed 5 oz., at 9 inches 6 oz., and at 18 inches 12 oz.; and on the other side of the middle, at 5 inches from the end 9 oz., and at the end a weight falsely marked 3 oz., when the stick was in equilibrium; what should it have been marked?

 Ans. 2 oz.
- 52. If a pair of steelyards weighing 5 lbs. will balance when supported 1 inch from the fulcrum, at what distances from the fulcrum must the 20, 25, 30, 35, and 40 lb. notches be made for a P weighing 3 lbs; the hook for the weight being $\frac{1}{2}$ an inch from the point of suspension?

 Ans. $1\frac{2}{3}$, $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{6}$ and 5 inches respectively.
- 53. A pine log 125 feet long, is found to balance, when supported 55 feet from the larger end; but when the support is removed to the middle, it requires 8000 lbs. on the small end to balance it; what is the weight of the log?

 Ans. 66,666‡ lbs.
- 54. A stick 38.9 inches long is found to balance itself when supported 19.15 inches from one end; but when suspended 11.1 inches from the same end, it requires 11½ oz. at that end, and ½ oz. 4 inches from the end, to produce an equilibrium. What is the weight of the stick?

 Ans. 15.73 oz.
- 55. What weight will be required to balance 500 lbs. on an inclined plane 20 feet long and 7 feet high?

 Ans. 175 lbs.
- 56. What will 30 lbs. balance on a plane, 15 feet long and 6 feet high?

 Ans. 75 lbs.
- 57. In rolling a barrel of cider weighing 300 lbs. into a wagon 4 feet above the ground, what force will be required, if the inclined plane or rails be 12 feet long?

 Ans. 100 lbs.

58. The length of an inclined plane is to its height as 12 to 5; what force parallel to the base would be required to raise 150 lbs. up the plane?

Ans. 68.75 lbs.

59. If in a cider press, the distance between the threads of the screw be an inch, and the strength of 3 men (150 lbs. each) be applied to the end of a lever 12 feet long; what will be the pressure?

Ans. 407,151 lbs.

- 60. A lever 5 feet long is fixed at right-angles, in a screw whose threads are 1 inch asunder. If this lever be urged by a force of 65 lbs. with what force will the screw press?

 Ans. 24504.35 lbs.
- 61. The sun is 95,000,000 miles from the earth, and is 354,936 times as heavy; where will be the centre of gravity between them?

Ans. 267.6 miles from the sun.

- 62. The moon is 240,000 miles from the earth, and $\frac{1}{4\pi}$ its weight; how far from the earth would they balance each other? Ans. 3158 miles nearly.
- 63. Four bodies, A, B, C, and D, weighing respectively 2, 3, 6, and 8 lbs.. are placed with their centres of gravity in a right line, at the distance of 3, 5, 7, and 9 feet from a given point; what is the distance of their common centre of gravity from that point?

 Ans. 7 1 ft.
- 64. If in a hydrostatic press, the large piston is 2 feet, and the small one ½ an inch in diameter, what pressure would be produced by the weight of a man (150 lbs.) on the end of a lever 10 feet long, having the small piston rod fastened one foot from the fulcrum? and how far will the large piston move, if the small one is forced down 6 inches?

Ans. 3,456,000 lbs. dist., 314 an inch.

- 65. Fishes are sometimes caught at the depth of 2,560 feet; what pressure must a fish with 100 square inches of surface, or about the size of a large shad, be capable of sustaining, in order to live at such a depth?

 Ans. 111111 lbs.
- 66. A bottle whose lateral surface contained 50 square inches, was let down into the sea to the depth of 3000 feet. What pressure would the sides of the bottle sustain, no allowance being made for the increased specific gravity of sea water?

Ans. 65104.166 lbs.

67. A mineral weighs 960 grains in air, and 739 grains in water; what is its specific gravity?

Ans. 4.344.

- 68. A block of marble when weighed in sea water is found to lose 641 lbs. 7 oz. of its weight, but when weighed in fresh water it loses only 625 lbs.; what is the specific gravity of sea water?

 Ans. 1.0263.
- 69. A person rows a boat directly across a river, at the rate of 4 miles an hour; if the breadth of the river is one mile, and the velocity of the current 3 miles per hour, how far will the boat move in crossing, and how far down the river will he land?

Ans. The boat moves 1½ miles; and he lands ½ m. down the river.

70. Three planets describe orbits, which are to each other as 15, 19, and 12; in times which are as 7, 3, and 5; what are their comparative velocities?

Ans. 225, 665, and 252.

71. Wishing to know the velocity of a musket ball weighing 1 oz., I suspended after the manner of a pendulum, a log of wood weighing 53 lbs. The ball

on entering the log gave it a motion of 2 feet per second; what was the velocity of the ball?

Ans. 1696 ft. per sec.

- 72. The light of the sun at the planet Jupiter is said to be about \$\frac{1}{27}\$th of that received at the earth; how far is Jupiter from the sun, taking the earth's distance 95,000,000 miles? Ans. 493, 634, 480 m.
- 7.73. A rock weighing 2700 tons, resting on three points, A., B., C., of a horizontal plane, whose distances are AB. 35; AC. 30, and BC. 25 feet; the perpendicular through the centre of gravity falling on the centre of the inscribed circle; required the weight on each point of support?

Ans. on A. 750, B. 900, and C. 1050 tons.

- 74. If a ball be projected downward, from a height of 1 mile, with a velocity of 3000 feet per second, how soon will it reach the earth? Ans. 1.744 sec.
- 75. A traveller in Kentucky, speaking of the Mammoth cave, says, "At the second waterfall, we amused ourselves by throwing down stones into the gulf below; and such was the enormous depth, that a minute elapsed between throwing the stone and hearing it strike the bottom." Supposing this account to be true, what was the depth?

Ans. 4 miles & fur. 130 yds.

- 76. A ball having been dropt from the top of a shot tower, was observed to strike the ground at the same instant that another ball did, which was dropt one second afterwards, from a window 64 feet below; what was the height of the tower?

 Ans. 100 ft.
- 77. A stone having been dropt from the top of a certain cliff, is found to be 9 times as long in reaching the ground as the sound is in returning; required the height.

 Ans. 1006.3 ft.

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- 78. A ball projected directly downward from the top of a building with a velocity of 48 feet per second, takes two seconds longer to reach the ground than the sound does to return; what is the height of the building?

 Ans. 70159 ft.
- 79. The length of Mauch-Chunk railway is 9 miles. If the angle of inclination be 1°, how long would it require a train of cars to descend from the summit by gravity; no allowance being made for friction?

 Ans. 6 min. 52½ sec.
- 80. A stone having been dropt into a deep well, it was observed that a string and plummet measuring just 18 inches from the point of suspension to the centre of oscillation, made 6 vibrations before the sound of its striking the bottom returned; what is the depth?

 Ans. 238.55 ft.
- 81. If in a pair of scales, a body weighs 900 lbs. in one scale, and only 400 in the other; what must be its true weight?

 Ans. 600 lbs.
- 82. The distance between the extremities of a false balance is 3 feet, and a body weighs 121 lbs. at one end and 144 at the other; how far from the middle is the point of suspension?

 Ans. ½3 of an inch.
- 83. What is the power of a screw, the threads of which are $\frac{1}{4}$ of an inch asunder, and the length of the lever 7 feet?

Ans, the weight = 2111 times the power.

84. An isosceles wedge whose width was 1 inch, and length 6, was inserted in the cleft of a rock. The pressure exerted perpendicularly against each side was 6600 lbs; what force applied to the back of the wedge would overcome the resistance? Ans. 556 lbs.

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- 85. Three bodies, A, B, and C, weighing respectively 5, 3 and 12 pounds, are so placed that AB = 8 feet, AC = 4 feet, and the angle BAC is a right-angle; what distance is the body C from the common centre of gravity?

 Ans. 2 ft.
- 86. What is the height of an inclined plane down which a square block will just roll, the length of the plane being 1 foot?

 Ans. 8,484 in.
- 87. What is the least height of a plane 30 inches long, if a rectangular block, 4 inches by 6 will roll down it?

 Ans. 24,96 in.
- 88. A wheel 2 feet in diameter, is placed upon an inclined plane, the length of which is to its height as 5 to 3; what must be the thickness of a rectangular—block, which when laid flat on the plane, shall be just sufficient to prevent the wheel from descending?

 Ans. 2.4 in.
- 89 If a rectangular block 5 inches in thickness, is just sufficient to prevent a wheel 4 feet 2 inches in diameter from descending an inclined plane; what is the inclination of the plane? Ans. L: H::5:3.
- 90. An inelastic body weighing 3 oz. and moving 10 feet per second, overtakes another weighing 2 oz. and moving 3 feet per second; what is the common velocity after impact?

 Ans. 7\frac{1}{4} ft. per sec.
- 91. An inelastic body weighing 7 oz. and moving 11 feet per second, strikes upon another at rest, weighing 15 oz.; required the velocity after impact?

Ans. 31 ft. per sec.

92. Two inelastic bodies meet when moving in opposite directions; one weighing 4 pounds with a velocity of 9, the other weighing 2 pounds with a velocity of 5 feet per second; what is the common velocity after they meet?

Ans. 4\frac{1}{3} ft. per sec.

- 93. A and B are two inelastic bodies, weighing 7 and 4 lbs. respectively, and moving in the same direction with velocities of 9 and 2 feet per second; when A overtakes B what velocity does it lose, and what does B gain?

 Ans. A $2\frac{e}{11}$ and B $4\frac{e}{11}$.
- 94. An inelastic body moving 7 feet per second, meets another moving 3 feet per second, and thus loses half its momentum: required the relative magnitudes of the two bodies?
- 95. There are two perfectly elastic bodies A and B, A weighing 10 lbs. and moving 8 feet per second, overtakes B weighing 6 lbs. and moving in the same direction, 5 feet per second; what are the velocities of A and B after impact?

Ans. A's 53, and B's 83.

96. A weighing 4 lbs. with a velocity of 6, meets B weighing 8 lbs., with a velocity of 4; if they are perfectly elastic, what will be their respective directions and velocities after impact?

Ans. A is thrown back with a velocity of 7¹/₄, and B with a velocity of 2³/₄.

- 97. A falling body was found to pass through 1th of the whole distance in the last second; how far did it fall?

 Ans. 3834.325 ft.
- 98. Upon a steeple 160 feet high, is a spire 50 feet; at the instant that a stone was dropt from the top of the steeple, another was projected vertically from the bottom of it, with a velocity sufficient to carry it to the top of the spire; how far from the bottom will these stones meet?

 Ans. 129.5 ft.
- 99. If a tube 30 feet long and an inch in diameter, be inserted in the end of a cylindrical vessel 6 feet high and 2 feet in diameter, what would be the

pressure on the sides of the vessel, when the tube is filled with water? also what force is exerted against the top?

Ans. against the sides 77755 lbs.; against the top 5890 lbs.

100. In an upright cylindrical vessel filled with water, whose height is 20 feet, how far from the bottom must a hole be made in order that the water may flow into a tub 6 feet distant, the top of the tub being on a level with the bottom of the vessel?

Ans. .4606 or 19.5394 ft.

101. Wishing to witness the effect of pressure in diminishing the bulk of air, I took a glass tube and bent it 9 inches from one end, in such a manner that both ends were parallel. The short end having been made air tight, I poured mercury into the other branch of the tube till it was filled 3 feet above the bend. Required the difference in the lengths of the two mercurial columns, and also the length of that part of the tube filled with air, the height of the barometer at that time being 30 inches?

Ans. diff. 31.4 inches; the length of the part

filled with air, 4.4 in.

102. Where, between the earth and moon, would a person weigh nothing; the mass of the earth being 75 times that of the moon?

Ans. 24843.2 miles from the moon.

103. If the matter in the earth were 4 times as great as it is, and the density the same, how much more would we weigh than we do now?

Ans. 1.586 times as much.

104. A traveller, when at Naples, 7 miles from Mount Vesuvius, observed fragments thrown from the crater of that volcano, at an elevation of 35° above the horizon; with what velocity were they thrown out?

Ans. 1287.1 feet per second.

- 105. At Philadelphia, the inclined plane on the Columbia rail-road, rises with an angle of about 5°; what weight, therefore, ought the rope to be capable of sustaining, to draw up a train of cars weighing 100 tons?

 Ans. 8.716 tons.
- 106. In drawing a load of two tons up a smooth inclined plane, whose angle of elevation is 10°, what force will be required?

 Ans. .3473 tons.
- 107. I shot an arrow, which remained in the air 4 seconds, and fell at the distance of 100 feet; with what angle of elevation was it propelled, with what velocity, and how high did it ascend? Ans. el. = 68° 40'; vel. = 68.71 ft. per sec.; h.=64.00 ft.
- 108. Leaving Cape Florida, we sailed N. 68° E., at the rate of 10 knots an hour, and in 12 hours reached the most western of the Bahamas. The gulf stream at this place, flows N. 10 E., with a velocity of 5 miles per hour; how far, and in what direction does this island lie from Cape Florida?

Ans. direct. = N. 49° 28' E.; dist. 160.1 miles.

109. A ferry-boat crosses a river $\frac{3}{4}$ of a mile wide in 45 minutes, the current running all the way at the rate of 3 miles an hour; at what angle with the direct course must the boat head up the stream, in order to move perpendicularly across? Also, in what ratio is the force required to move the boat increased, in consequence of the current?

Ans. 71° 34'; ratio, 1: 3.162.

110. A shot dropt from the topmast of a ship, sailing at the rate of 12 miles per hour, reached the deck in 3 seconds. Required the length and nature of the curve described by the body in its fall?

Ans. A parabola; abscissa 144; ordinate 52.8; curve 174.458 ft. in length.

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- 111. The elevation of an inclined plane is 30°. A body in falling from the top to the bottom of it, acquires a velocity of 576 feet per second; required the length of the plane?
- 112. How long will a body be in descending the last half of a plane, whose height is one mile, and angle of elevation 1'? Ans. 5 hrs. 4 min. 48.7 sec.
- 113. At Alphack, in Switzerland, there was formerly a celebrated slide for conveying timber trees from Mount Pilatus to Lake Luzerne, a distance of 8 miles, whence they were transported down the The slide consisted of an inclined plane, formed of logs in the shape of a trough, inclined to the horizon on an average, at an angle of 3° 14'; into this trough the trees are launched, and descend by the force of gravity. In what time will a tree descend the whole distance, no allowance being made for friction? Ans. 3 min. 36 sec.
- 114. Two inclined planes have a common height of 75 feet; the elevation of one of them is 50°, and of the other 20°; with what velocity must a body be projected from the top of the former, that it may just rise to the top of the latter; and what will be the whole time of its ascending and descending through the two planes? Ans. vel. \Rightarrow 0: time 9.1565 sec.
- 115 A large vessel 10 feet high, being kept constantly full of water; if 9 small circular holes, each 1 of an inch in diameter, be opened in its perpendicular side at every foot of the depth, it is required to find the several distances to which the water will spout on the horizontal plane of the base, and the quantity of water discharged by all of them in ten minutes; supposing the velocity of the water to be the same in all parts of the holes. Ans. distances are 6, 8, 9.165, 9.8, 10, 9.8, 9.165, 8 and 6;

quantity of water 123.88 ale gallons.

- 116. Supposing the perpendicular height of a cylinder to be twice the diameter of the base, what is the limit of its angle of inclination, before it is in danger of falling?

 Ans. 63° 26' to the horizon.
- 117. There are two inclined planes, each 20 inches in length; the inclination of one is such, that a pentagonal block will just roll down it; and that of the other such, that a rectangular block 4 inches by 3, will also roll: required the difference of the heights of the two planes?

 Ans. 4.244 inches.
- 118. Three bodies, A, B and C, weighing respectively, 3, 2, and 1 lbs., have their centres of gravity joined by the lines, AB, BC, CA; of which AB = 5 feet, BC = 4, CA = 2; what is the distance of the centre of gravity from the body C?

Ans. 1.394 ft.

- 119. What is the diameter of a wheel, which a rectangular block 2 inches in thickness, will just prevent from rolling down an inclined plane, whose length is to its height, as 7 to 8? Ans. 41.45 inches.
- 120. The arms of a lever are to each other as 7, 9, and it is acted on obliquely by two forces: the force P applied at the extremity of the longer arm, is inclined to it at an angle of 50°, and p at the shorter, at an angle of 80°; what is the proportion between the forces, when the lever is in equilibrium? Ans. P = p.
- 121. I took a brass rod 4 feet long, and bending it in the middle, hung it up by a string; on one end I placed a weight of 4 lbs.; on the other, one of 10 lbs., when the former end was found to be exactly parallel to the horizon: how much did I bend it?

Ans. 66° 25'.

122. The arms of a bent lever are 6 and 8 feet

respectively, and contain an angle of 120°; on the shorter arm there is placed 10 lbs., on the other 7; what inclination will the two arms have to the horizon?

Ans. longer 26° 34′; shorter 35° 26′.

- 123. A spherical body weighing 400 lbs. is supported between two inclined planes. The inclination of one to the horizon is 30°, and that of the other is 60°; what is the perpendicular pressure against each? Ans. on the first 346.4, on the other 200 lbs.
- 124. Over two fixed pulleys, which are on a level with each other, there is placed a rope having a weight of 10 lbs. on each end; what angle would the rope form in the middle, if a weight of 8 lbs. were hung there?

 Ans. 132° 50'.
- 125. A body is projected at an angle of 15° above the horizon, with the velocity of 140 feet per second; how far was it thrown?

 Ans. 306.25 ft.
- 126. A body is projected at an angle of 60° with the horizon, and descends at the distance of 100 feet; with what velocity was it thrown, and what its greatest altitude?

 Ans. V. = 60.79, Al. = 43.305.
- 127. A gun was fired at an elevation of 50°, and the shot struck the ground at the distance of 4898 feet; with what velocity did it leave the gun, and how long was it in the air?

Ans. V. = 398.5 ft. per sec. T = 19.1 sec.

- 128. With an elevation of 40°, a ball is thrown 5000 feet; how far would it be thrown with an elevation of 30°?

 Ans. 4397 ft.
- 129. A walking stick 39.4 inches long, when suspended in the middle, required 2 oz. at the large end, \(\frac{1}{2}\) oz. at the small end, and an unknown weight 8.4 inches from the middle, to balance it; but upon mov-

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ing the last weight 2 inches nearer the middle, it was balanced by placing the 2 oz. weight 14 inches from the middle; then taking off the ½ oz. weight, and moving the point of suspension .8 inches nearer the large end, it was again in a state of equilibrium. What was the weight of the stick, and also that of the unknown weight?

Ans. unknown weight, 5.7 oz., and weight of the stick, 4.6125 oz.

130. An island of ice rises 30 feet out of the water, and its upper surface is a circular plane, containing of an acre. On the supposition that the mass is cylindrical; required its weight and depth below the water; the specific gravity of sea water being 1.0263, and that of ice .92.

Ans. weight 242900 tons, depth 259.64 ft.

- 131. A cone whose height is 6 inches, is immersed in water, with its vertex downwards. Its specific gravity being .3, what part of the axis will be immersed?

 Ans. 4.017 in.
- 132. With what rapidity must the earth revolve on its axis, that a body at the equator should weigh nothing; the present centrifugal force being 115 part of gravity; and what must it be to lose 1 and 1 of its present weight?

 Ans. 1 h. 24 m. to lose all.

1 h. 59 m. to lose ½. 2 h. 26 m. to lose ‡.

- 133. In what time must the earth turn on its axis, that a falling body may descend only 10 feet in the first second; and how soon to descend only 1 foot in the same time? Ans. 2 h. 18 m., and 1 h. 27 m.
- 134. In raising a ladder against the side of an upright wall, according to the common method of securing the large end, and then beginning at the small

end to raise, what weight would a man, who reaches 6 feet, sustain with his hands when the ladder makes an angle of 35° with the ground; if the ladder be 25 feet long, weigh 60 lbs., and balance 10 feet from the large end?

Ans. if the force is applied perpent to the ladder 46.98 lbs.

135. On a hill-side ascending uniformly above a horizontal level at an angle of 10° 20', a ball was fired at an angle of elevation of 34°, and with a velocity of 401 feet per second; what was the range on the hill-side when the gun was directed up the hill, and what when directed down?

Ans. 3456 and 6016.

136. Hiero, king of Syracuse, ordered his jeweller to make him a crown of gold containing 63 oz. The artist attempted fraud, by substituting a certain portion of silver, which being suspected, the king appointed Archimedes to examine it. Archimedes put it into water, and found that it raised the fluid 8.2245 cubic inches; and having found that an inch of gold weighed 10.36 oz., and that of silver 5.85 oz., he discovered what part of the gold had been purloined; required the quantity.

Ans. 28.8 oz.

137. A heavy body dropped by an aeronaut, from a great height in the air, was observed to be twice as long in descending to the earth, as the sound of its striking was in returning: required the height of the balloon; the velocity of the body when it reached the ground; the comparative gravities of the bodies at the balloon and surface of the earth; and the greatest distance the aeronaut would be able to see on the earth's surface around him, supposing the diameter of the earth to be 8000 miles.

Ans. height 20377.5 feet; velocity 1142 feet per sec.; gravity at balloon; gravity at earth::1:1.00193, and distance 175.5 miles.

138. There is a river, the banks of which are perpendicular cliffs, whose heights are 1296 and 576 feet respectively. Now it has been observed, that two heavy bodies being let fall from the top of each, at the same time, the sound of their striking the water reached the summit of the higher at the same instant. Required the breadth of the river?

Ans. 4541 feet nearly.

- 139. A cylindrical diving bell, 12 feet in length, was let down into the sea until its lower extremity was 56 feet below the surface: it is required to determine the height the water rose within the bell, the pressure of the air at that time being equal to 32 feet of water.

 Ans. 7.245 ft.
- 140. Suppose I throw a stone into a well, and observe a pendulum 12 inches long, make 20 vibrations, from the moment of dropping to the return of the sound to my ear; required the depth of the well?

 Ans. 1520 ft.
- 141. A pendulum vibrates as often in a minute as it is inches in length. Quere what that length is?

 Ans. 52.02 in.
- 142. A clock maker having put a new pendulum to an old time piece, found it gained 5 minutes in 12 hours; but after he had lengthened it two inches, it lost 5 seconds per hour; what ought to be the true length of the pendulum, and how often will it vibrate per minute? Ans. length 121 inches; number of vibrations 34.44.
- 143. Travelling on a road running directly west, at the rate of 6 miles an hour, I observed the wind appeared to strike me from the N. W., but having occasion to stop, I then found that it actually came from a point 10° more to the north. Required the velocity of the wind?

 Ans. 24.43 miles an hour.

- 144. Three men undertake to pull down an old stone column, whose height is 30 feet. For this purpose they fasten 3 ropes, each 50 feet long, to a ring in the top; and each standing in such a position that the ropes make equal angles of 30° with each other, pulls with a force of 200 pounds. Now, if this force was just sufficient to overturn the column, it is required to find how much of their strength might have been saved, by all three pulling on a rope passed over a fixed pulley, on a level with the top of the pillar.

 Ans. 187.04 lb. lost.
- 145. If a solid rock weighing 1000 tons, rests on a horizontal plane, at the angular points of an equilateral triangle, and the vertical line through the centre of gravity meets the same plane at the distances of 35, 40 and 45 feet respectively, from those angles; required the weight resting on the several points of support?

 4/6 Ans. 390.54 tons, at 35 from centre.

 337
 314.43
 at 40 from centre.

3 3 7 295.03 at 45 from centre. of gravity.

- 146. If a pendulum, which is 60 <u>inches</u> long from the point of suspension to the centre of oscillation, vibrates through an arc of 15°, how many vibrations will it perform in 24 hours? the distance which a heavy body falls in the first second, being reckoned at 16 feet.

 Ans. 20063.
- 147. If we desire to construct a trough, along which a heavy body will descend by its own gravity in the shortest time possible, through a vertical distance of 50, and an horizontal one of 78.54 feet; what will that time be? what the length of the trough? and what the time of passing from one of

the extreme points to the other, along a single inclined plane?

Ans. least time 2.776 seconds.

Length of trough 100 ft. 20

Time on inclined plane 3568 sec.

148. What must be the length of a pendulum which would vibrate seconds in an arc of 5°, on the surface of the planet Jupiter; the gravity there being to that on the surface of the earth as 2.3287 to 1; and their day being 9 hours 56 minutes of our time?

Ans. 15.6 inches.

149. Considering the inclined plane near Peters' island to be 180 feet in vertical descent, and half a mile in length, how long would a car require to descend by the force of gravity, from the top to the bottom, allowing the friction to be equal to $\frac{1}{100}$ part of the weight; and how far would it move with the last acquired velocity, while a pendulum 10 feet long makes one vibration through an arc of 18°?

Ans. time of descent 51 sec. distance 180 ft. / 8 2 /f.

150. Suppose three observers, A, B, C, whose distances asunder are AB 100 yards, BC 118 yards, AC 169 yards, are prepared to take angles of altitude of a balloon, at the instant when the aeronaut by a preconcerted signal, informs them that the mercury in his barometer, which on the ground stands at 30 inches, has fallen to 29.5 inches; and then find those altitudes at A 53° 53', at B 46° 40', at C 52° 56'; what will be the height when the mercury has fallen to 14 inches? no allowance being made for change of temperature.

Ans. 6773 yards.

And. 6 857 or 10360 4dl.

ERRATA.

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